Exponential Rates of Change

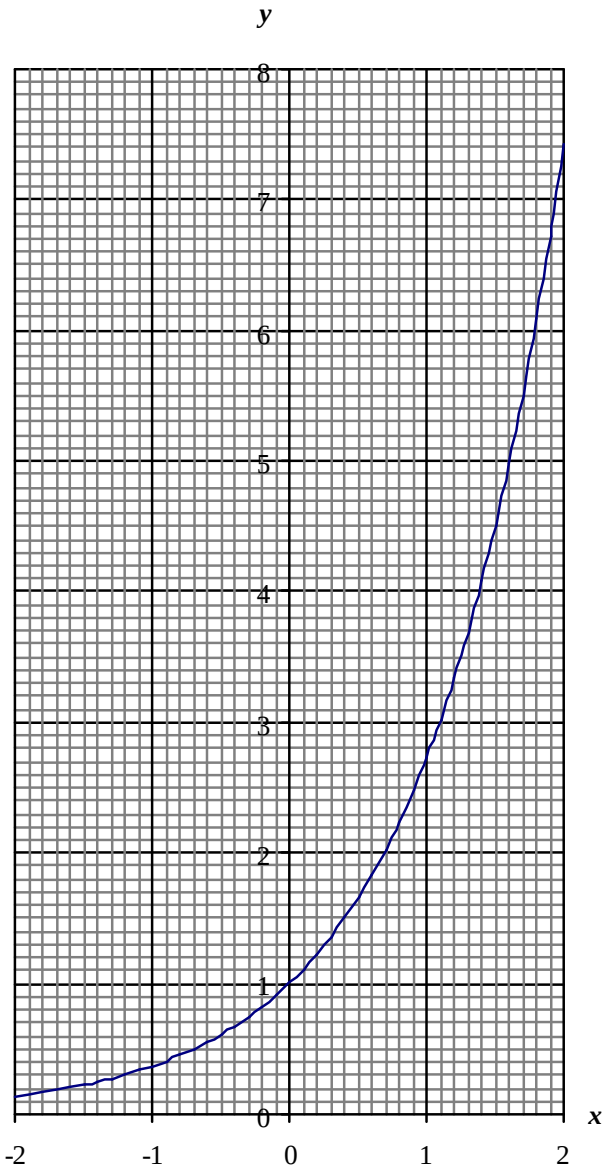
Worksheet A

Use your calculator to complete the $y = e^x$ row in the table below, giving values to 1 d.p.

x	-2	-1	0	1	2
$y = e^x$					
Gradient					

Check that your values agree approximately with points that lie on the curve shown below.

Graph of $y = e^x$



Draw tangents to the curve at the points given in the table.

Find the gradient of each tangent and write the value, correct to 1 d.p, in the table.

Compare the values in the last two rows of the table.
Do you notice anything?

Exponential Rates of Change

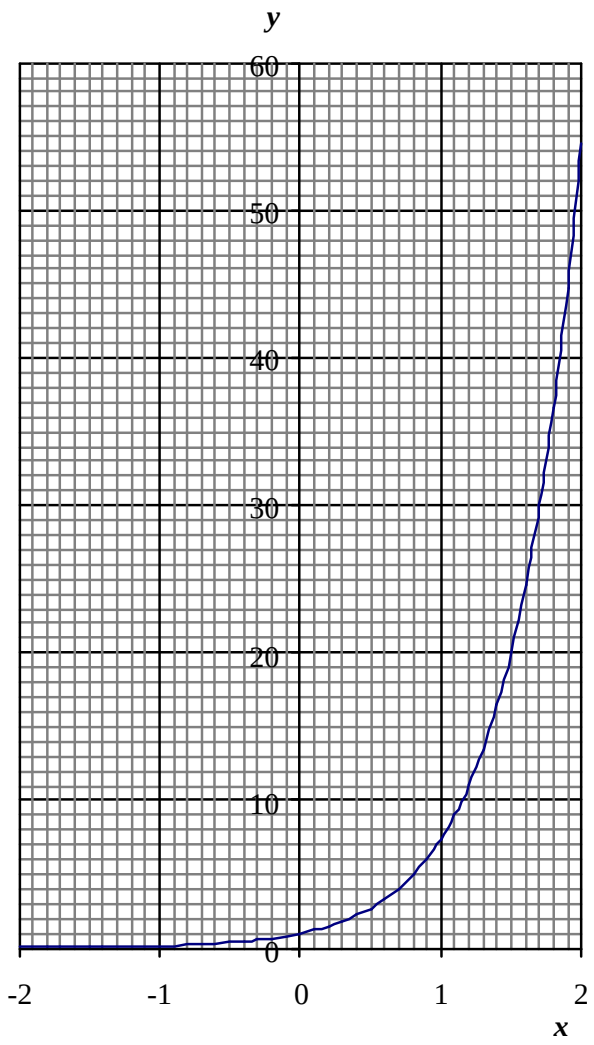
Worksheet A

Use your calculator to complete the $y = e^{2x}$ row in the table below, giving values to 1 d.p.

x	-2	-1	0	1	2
$y = e^{2x}$					
Gradient					

Check that your values give points that lie on the curve shown below.

Graph of $y = e^{2x}$



Draw tangents to the curve at the points given in the table.

Find the gradient of each tangent and write the value, correct to 1 d.p, in the table.

Compare the values in the last two rows of the table.
Do you notice anything?

Exponential Rates of Change

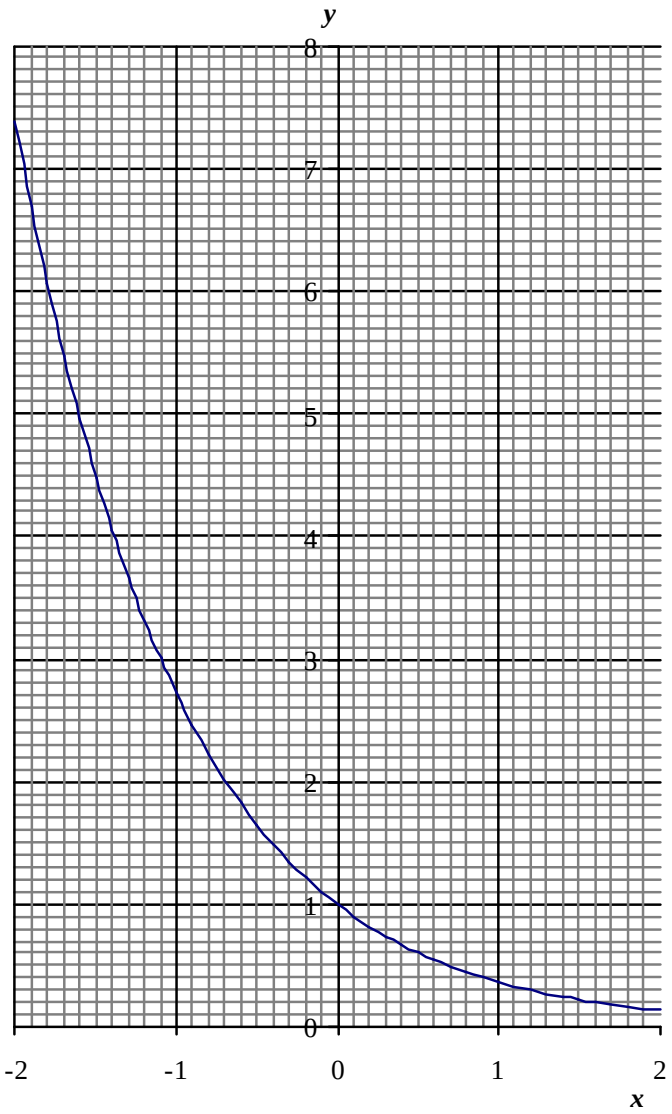
Worksheet A

Use your calculator to complete the $y = e^{-x}$ row in the table below, giving values to 1 d.p.

x	-2	-1	0	1	2
$y = e^{-x}$					
Gradient					

Check that your values agree approximately with points that lie on the curve shown below.

Graph of $y = e^{-x}$



Draw tangents to the curve at the points given in the table.

Find the gradient of each tangent and write the value, correct to 1 d.p, in the table.

Compare the values in the last two rows of the table.
Do you notice anything?

Exponential Rates of Change

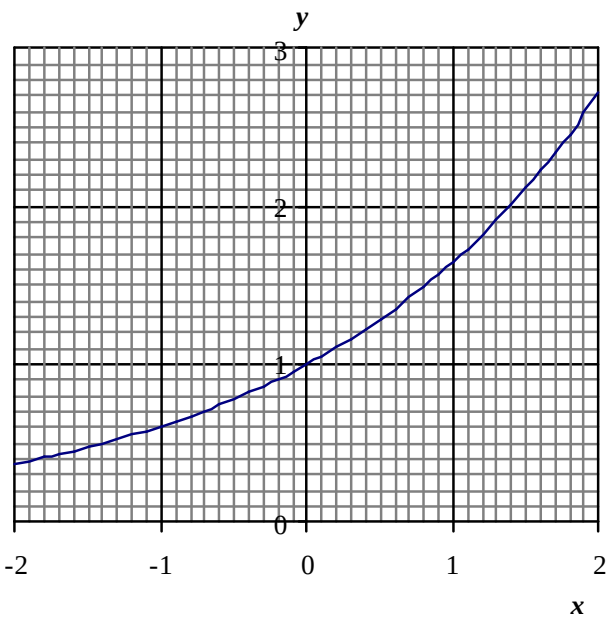
Worksheet A

Use your calculator to complete the $y = e^{0.5x}$ row in the table below, giving values to 1 d.p.

x	-2	-1	0	1	2
$y = e^{0.5x}$					
Gradient					

Check that your values agree approximately with points that lie on the curve shown below.

Graph of $y = e^{0.5x}$



Draw tangents to the curve at the points given in the table.

Find the gradient of each tangent and write the value, correct to 1 d.p, in the table.

Compare the values in the last two rows of the table.
Do you notice anything?

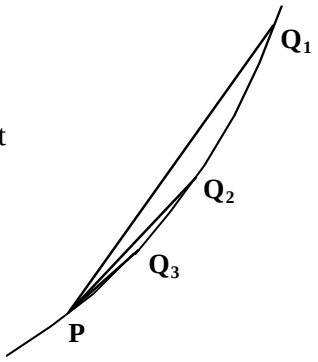
Exponential Rates of Change

Worksheet B

Gradients

Finding the gradient of curves by drawing tangents by hand is not a very accurate method. Better results can be achieved by calculation.

The sketch shows a point P on a curve. Suppose that Q_1 is a second point on the curve near to P. The co-ordinates of P and Q_1 can be used to find the gradient of the chord PQ_1



Other points Q_2 and Q_3 that lie on the curve even nearer to P are also shown on the sketch. Note that the nearer the point Q is to P, the nearer the gradient of PQ is to the gradient of the tangent at P.

In general, the gradient at a point P where $x = a$, on the curve $y = f(x)$ is given by:

$$\text{gradient} \approx \frac{f(a+h) - f(a)}{h} \qquad \text{where } h \text{ is a small increment}$$

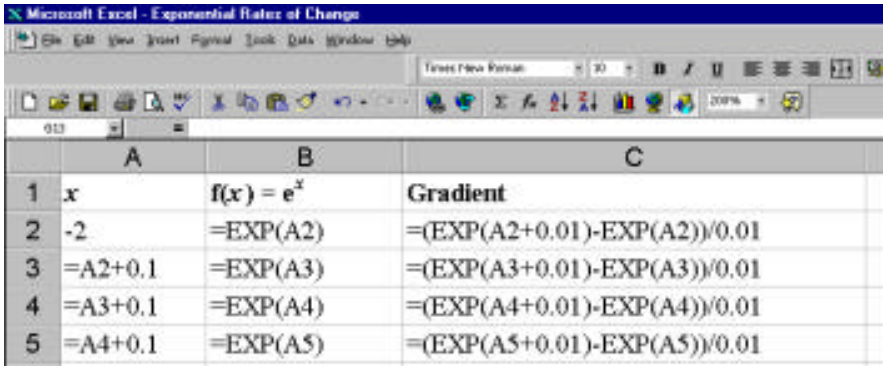
(In terms of the sketch h represents the difference in the x co-ordinates at P and Q and $f(a+h) - f(a)$ the difference in the y co-ordinates of P and Q.)

The gradient function of $y = e^x$

A spreadsheet can be used to perform the calculations needed to estimate the gradient at a number of points on a curve.

The spreadsheet below gives formulae that can be used to estimate gradients on the curve $y = e^x$.

The formulae in column A work out x co-ordinates for the curve at intervals of 0.1 starting with $x = -2$
The formulae in column B work out the corresponding y co-ordinates.
The formulae in column C estimate the gradient of the curve at each point using an increment of 0.01.



	A	B	C
1	x	$f(x) = e^x$	Gradient
2	-2	=EXP(A2)	=(EXP(A2+0.01)-EXP(A2))/0.01
3	=A2+0.1	=EXP(A3)	=(EXP(A3+0.01)-EXP(A3))/0.01
4	=A3+0.1	=EXP(A4)	=(EXP(A4+0.01)-EXP(A4))/0.01
5	=A4+0.1	=EXP(A5)	=(EXP(A5+0.01)-EXP(A5))/0.01

Copy these formulae onto a spreadsheet, using 'fill down' to extend the results to $x = 2$.

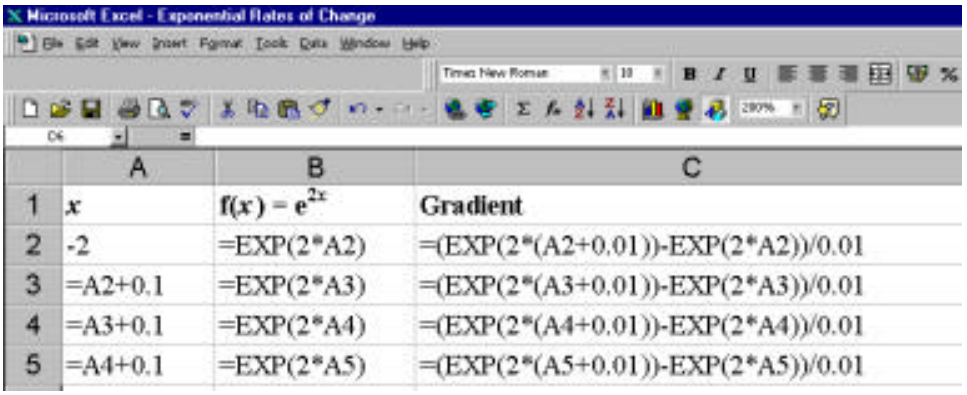
Compare the values found in columns B and C. What do you notice?

Exponential Rates of Change

Worksheet B

The gradient function of $y = e^{2x}$

The spreadsheet below shows formulae that can be used to estimate gradients on the curve $y = e^{2x}$.



	A	B	C
1	x	$f(x) = e^{2x}$	Gradient
2	-2	=EXP(2*A2)	=(EXP(2*(A2+0.01))-EXP(2*A2))/0.01
3	=A2+0.1	=EXP(2*A3)	=(EXP(2*(A3+0.01))-EXP(2*A3))/0.01
4	=A3+0.1	=EXP(2*A4)	=(EXP(2*(A4+0.01))-EXP(2*A4))/0.01
5	=A4+0.1	=EXP(2*A5)	=(EXP(2*(A5+0.01))-EXP(2*A5))/0.01

Copy these formulae onto another worksheet and use 'fill down' to extend the results to $x = 2$.

Compare the values found in columns B and C. What do you notice this time?

Use the spreadsheet to draw graphs of $y = e^{2x}$ and its gradient function on the same axes. Compare the curves and write down what you notice.

Gradient functions of other exponential functions.

Make a copy of the worksheet you used for $y = e^{2x}$.

Find values for $y = e^{0.5x}$ and its gradient function by replacing '2' in cells B1, B2 and C2 by '0.5' (leaving A2 unchanged). Use 'fill down' to change the other cells in columns B and C and extend the table to $x = 2$.

Again compare the values in columns B and C and draw graphs of $y = e^{0.5x}$ and its gradient function on the same axes. Write down what you notice.

Repeat this process for $y = e^{-x}$ and other exponential functions of the form $y = e^{kx}$ where k is any constant

Can you say anything in general about the gradient function of $y = e^{kx}$?

Investigate the gradient functions of functions of the form $y = ae^{kx}$, $y = e^{kx + c}$ and $y = ae^{kx + c}$ where a, k and c are constants.

Teacher Notes

Unit Advanced Level, Modelling with calculus

Skills used in this activity:

- finding gradients by drawing tangents
- finding gradients in Excel using a small increment.

Preparation

Students need to know how to find the gradient of a curve by drawing a tangent and how to enter spreadsheet formulae and use 'fill-down' in Excel. They will each need a copy of one of the Worksheet A pages (P1 – 3) and a copy of both of the Worksheet B pages (P5 – 6).

Notes on Activity

This activity can be used to introduce the differentiation of exponential functions. It is recommended that you divide students into groups, each finding gradients for just one of the graphs and then pool the results. The second part of the activity requires access to Excel. Students are given spreadsheet formulae that can be used to estimate gradients at points on the curves $y = e^x$ and $y = e^{2x}$ using a small increment. They are asked to investigate the gradient functions of these and other exponential functions, using the numerical values given by the spreadsheet and graphs drawn from them.

Answers

x	-2	-1	0	1	2
$y = e^x$	0.1	0.4	1.0	2.7	7.4
Gradient	0.1	0.4	1.0	2.7	7.4

Gradient = e^x

x	-2	-1	0	1	2
$y = e^{2x}$	0.0	0.1	1.0	7.4	54.6
Gradient	0.0	0.3	2.0	14.8	109.2

Gradient = $2e^{2x}$

x	-2	-1	0	1	2
$y = e^{-x}$	7.4	2.7	1.0	0.4	0.1
Gradient	-7.4	-2.7	-1.0	-0.4	-0.1

Gradient = $-e^{-x}$

x	-2	-1	0	1	2
$y = e^{0.5x}$	0.4	0.6	1.0	1.6	2.7
Gradient	0.2	0.3	0.5	0.8	1.4

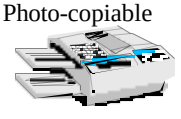
Gradient = $0.5e^{0.5x}$

The following pages give some of the graphs students are asked to draw.
These could be copied onto OHTs for use in class discussion.

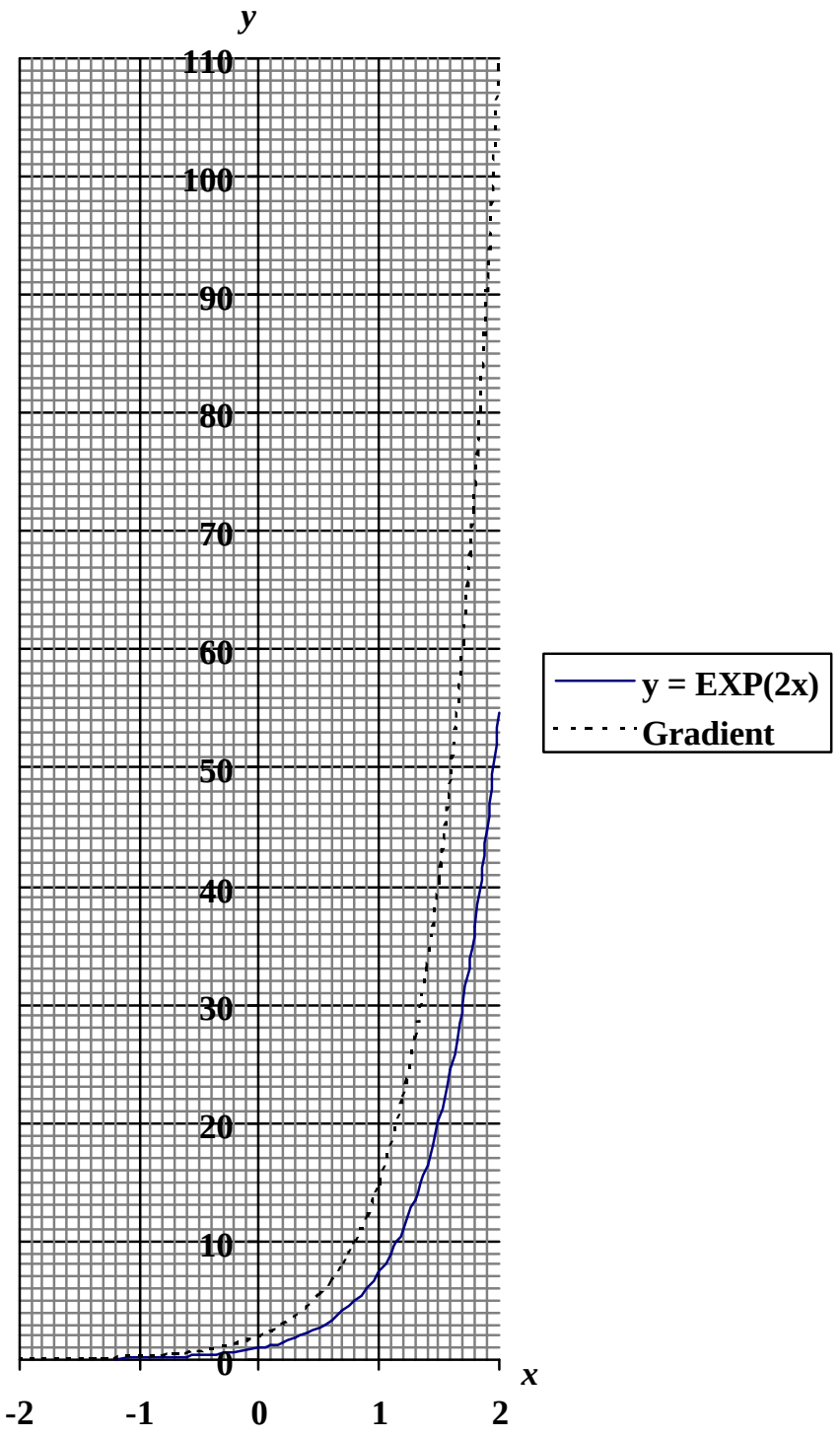
The results that can be found from this activity are listed below:

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(e^{2x}) = 2e^x \qquad \frac{d}{dx}(e^{0.5x}) = 0.5e^{0.5x} \qquad \frac{d}{dx}(e^{-x}) = -e^{-x}$$

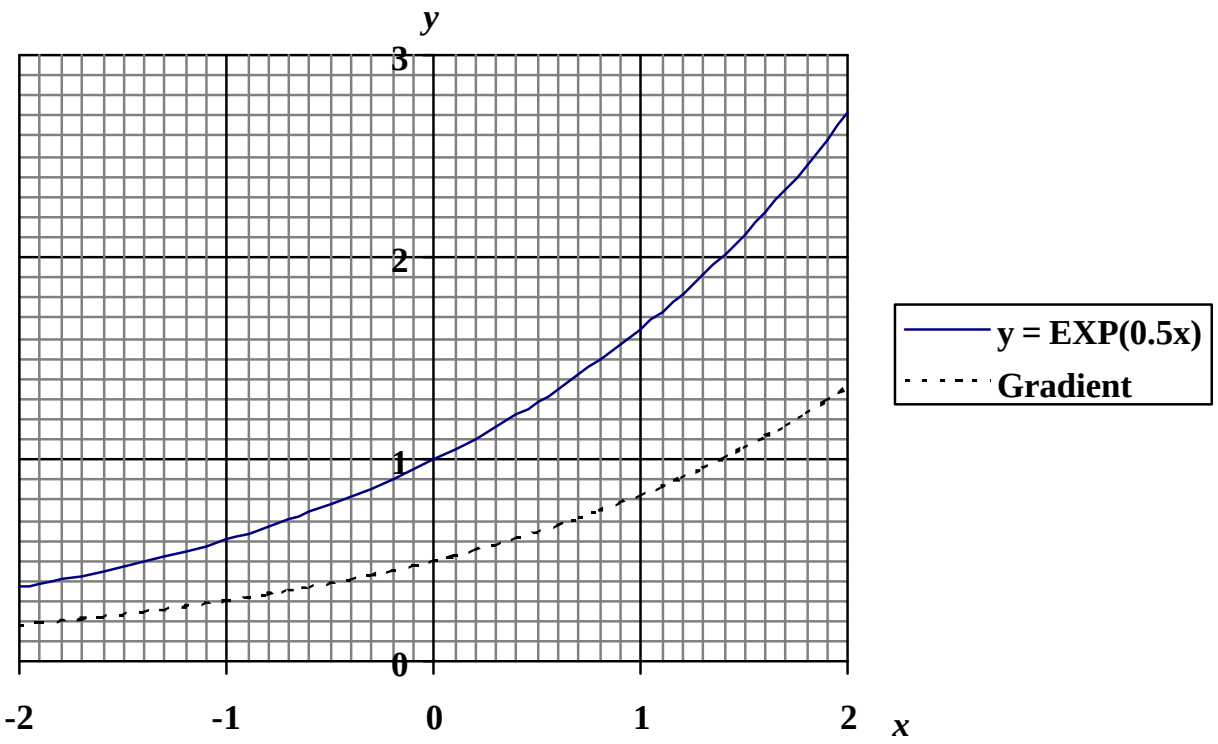
$$\frac{d}{dx}(e^{kx}) = ke^{kx} \qquad \frac{d}{dx}(ae^{kx}) = kae^{kx} \qquad \frac{d}{dx}(e^{kx+c}) = ke^{kx+c} \qquad \frac{d}{dx}(ae^{kx+c}) = kae^{kx+c}$$



Graph of $y = e^{2x}$ and its gradient function



Graph of $y = e^{0.5x}$ and its gradient function



Graph of $y = e^{-x}$ and its gradient function