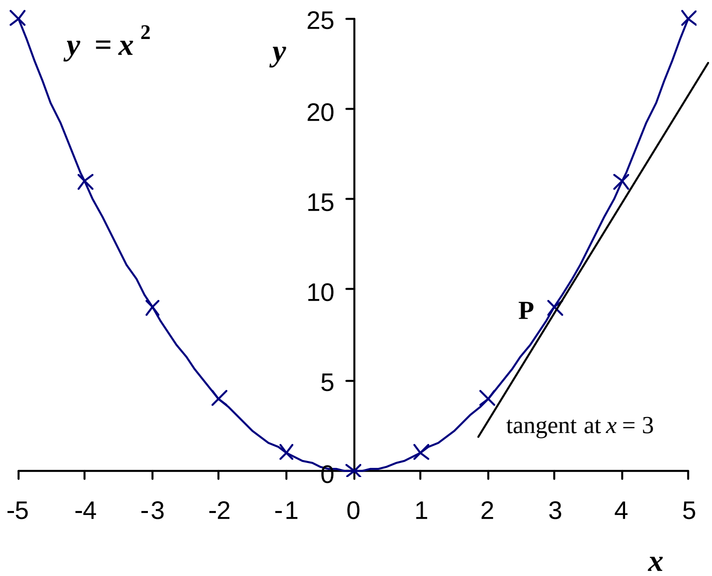
Gradients

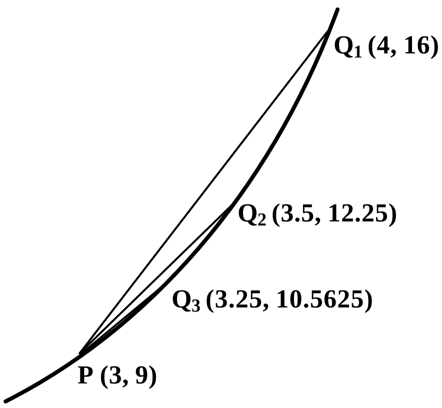
The gradient of a curve has different values at different points.

The sketch shows the graph of $y = x^2$. When x is negative the gradient is negative, because y is decreasing as x increases. When x is positive the gradient is positive and increases as x increases i.e. the curve becomes steeper.

A tangent has been drawn at the point, P, on the curve where $x = 3$. The gradient of the curve at this point is the gradient of this tangent.

The gradient can be found from a hand-drawn graph but this method would give an estimate rather than an accurate result.





Look at the second sketch.

It shows the section of the curve between the points P(3, 9) and Q_1(4, 16).

The gradient of the straight line PQ_1 is given by:

$$\text{gradient of } PQ_1 = \frac{\text{difference in } y \text{ values}}{\text{difference in } x \text{ values}}$$

$$= \frac{16-9}{4-3} = 7$$

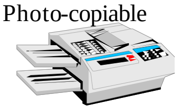
Now suppose another point, nearer to P, is used rather than Q_1
At the point Q_2, the x co-ordinate is 3.5, and the y co-ordinate is $3.5^2 = 12.25$

$$\text{Gradient of } PQ_2 = \frac{12.25-9}{3.5-3} = \frac{3.25}{0.5} = 6.5$$

Repeating this at the point Q_3, where the x co-ordinate is 3.25, and the y co-ordinate is $3.25^2 = 10.5625$

$$\text{Gradient of } PQ_3 = \frac{10.5625-9}{3.25-3} = \frac{1.5625}{0.25} = 6.25$$

As the point Q moves nearer to P, the gradient of PQ becomes nearer to the gradient of the tangent at P.
Carrying out this process a number of times is time-consuming because of the arithmetic involved.
A spreadsheet can be used to perform the calculations and give results more quickly.



Using the spreadsheet to find gradients

The ***x* squared** worksheet has been set up to show the gradients calculated on page 1.

	A	B	C	D
1	**Point**	***x***	***y = x²***	**Gradient of PQ**
2	P	3	9	
3	Q_1	4	16	7
4	Q_2	3.5	12.25	6.5
5	Q_3	3.25	10.5625	6.25

The spreadsheet formulae that have been used are given below. Look at them carefully.

	A	B	C	D
1	**Point**	***x***	***y = x²***	**Gradient of PQ**
2	P	3	= B2^2	
3	Q_1	= B2 + 1	= B3^2	= (C3 –-C2)/(B3 – B2)
4	Q_2	= (B2 + B3)/2	= B4^2	= (C4 –-C2)/(B4 – B2)
5	Q_3	= (B2 + B4)/2	= B5^2	= (C5 –-C2)/(B5 – B2)

gradient is difference in *y* values divided by difference in *x* values
Absolute references for cells C2 and B2 mean the co-ordinates of P are used each time.

The *x* co-ordinate used for Q is halfway between the *x* co-ordinate of P and the previous *x* co-ordinate.

squaring the *x* value gives the *y* value

Use '*fill-down*' to extend the table as far as Q_{10}.
The values calculated by the spreadsheet are given below.

	A	B	C	D
1	**Point**	***x***	***y = x²***	**Gradient of PQ**
2	P	3	9	
3	Q_1	4	16	7
4	Q_2	3.5	12.25	6.5
5	Q_3	3.25	10.5625	6.25
6	Q_4	3.125	9.765625	6.125
7	Q_5	3.0625	9.37890625	6.0625
8	Q_6	3.03125	9.188476563	6.03125
9	Q_7	3.015625	9.093994141	6.015625
10	Q_8	3.0078125	9.046936035	6.0078125
11	Q_9	3.00390625	9.023452759	6.00390625
12	Q_{10}	3.001953125	9.011722565	6.001953125

As Q approaches P the gradient of PQ approaches the value 6.

This is the gradient of the tangent to the curve at P.

The gradient of $y = x^2$ at the point (3, 9) is 6.

On this worksheet the numerical values in the cells giving *x*, *y* and gradient values are all determined from the value entered in cell B2. This means that the gradient at any other point on $y = x^2$ can be found by simply replacing the value in B2 by the *x* co-ordinate of the new point.

Enter the value 2 in cell B2.
You should find that the values in the other cells change to give you the gradient of $y = x^2$ at the point (2,4).
The gradient is 4.

The values of the gradients found so far are given in the table.

Use the spreadsheet to complete the table.

What is the relationship between the gradient and the *x* co-ordinate?

y = x²

Point	*x* co-ordinate	gradient
(– 4, 16)	– 4	
(– 3, 9)	– 3	
(– 2, 4)	– 2	
(– 1, 1)	– 1	
(0, 0)	0	
(1, 1)	1	
(2, 4)	2	4
(3, 9)	3	6
(4, 16)	4	

Gradient = ……………………………………….

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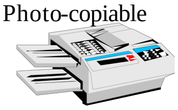

Gradient functions

For the curve $y = x^2$ the gradient function is $2x$

The ***x* cubed** worksheet has been set up to find the gradient of the curve $y = x^3$ at the point (1,1)
Look carefully at the spreadsheet functions that have been used to start the table.
Compare them with those that were used for $y = x^2$ (given on page 2).

Use '*fill-down*' to extend the table as far as Q_{10}.
This will show that the gradient of the curve $y = x^3$ at the point (1,1) is 3.

Use the spreadsheet to complete the table.

What is the gradient function for $y = x^3$?

$y = x^3$

Point	*x* co-ordinate	gradient
(– 4, – 64)	– 4	
(– 3, – 27)	– 3	
(– 2, – 8)	– 2	
(– 1, – 1)	– 1	
(0, 0)	0	
(1, 1)	1	3
(2, 8)	2	
(3, 27)	3	
(4, 64)	4	

Gradient function = ……………………….

Set up a new worksheet to find the gradient of the curve $y = x^4$ at the point (1,1)

Use your worksheet to complete the table.

What is the gradient function for $y = x^4$?

$y = x^4$

Point	*x* co-ordinate	gradient
(– 4, 256)	– 4	
(– 3, 81)	– 3	
(– 2, 16)	– 2	
(– 1, 1)	– 1	
(0, 0)	0	
(1, 1)	1	
(2, 16)	2	
(3, 81)	3	
(4, 256)	4	

Gradient = …………………………………….

Complete the 2nd and 3rd rows of this table :

What do you think is the gradient function of $y = x^5$?

Use a new worksheet to check your answer.

Equation of Curve	Gradient function
$y = x^2$	$2x$
$y = x^3$	
$y = x^4$	
$y = x^5$	

Teacher Notes

Unit Advanced Level, Modelling with calculus

Skills used in this activity:

- using Excel to find gradients of curves by finding gradients of chords

Preparation

Students need to know how to find the gradient of a straight line and how to enter spreadsheet formulae and use ‘fill-down’ in Excel. They will each need a copy of pages 1 to 3 and the Excel speadsheet, Gradients.xls.

Notes on Activity

The first four slides of the powerpoint presentation can be used to introduce differentiation. (A shortened version of this is given on Page 1.) It is suggested that students then use the spreadsheet. The first worksheet is set up to find the gradients of a series of chords from the point P(3,9) on $y = x^2$. Fill-down will enable students to see the gradient approaching the value 6 as Q approaches P. Students are then asked to find the gradient at other points on the curve and hence the relationship between the gradient and the value of x. The second worksheet is similarly set up for $y = x^3$. After setting up their own worksheet for $y = x^4$, students are asked to look for a pattern in the gradient functions and suggest, then check, the gradient function for $y = x^5$. After using the spreadsheet, slides 5 to 8 of the powerpoint presentation can be shown. These show how a general gradient function is found for $y = x^2$, then summarises the results for other powers before finding the gradient function for a cubic curve.

Answers

$y = x^2$

Point	x co-ordinate	gradient
(– 4, 16)	– 4	**– 8**
(– 3, 9)	– 3	**– 6**
(– 2, 4)	– 2	**– 4**
(– 1, 1)	– 1	**– 2**
(0, 0)	0	**0**
(1, 1)	1	**2**
(2, 4)	2	4
(3, 9)	3	6
(4, 16)	4	**8**

Gradient = $\mathbf{2x}$

$y = x^3$

Point	x co-ordinate	gradient
(– 4, – 64)	– 4	**48**
(– 3, – 27)	– 3	**27**
(– 2, – 8)	– 2	**12**
(– 1, – 1)	– 1	**3**
(0, 0)	0	**0**
(1, 1)	1	3
(2, 8)	2	**12**
(3, 27)	3	**27**
(4, 64)	4	**48**

Gradient function = $\mathbf{3x^2}$

$y = x^4$

Point	x co-ordinate	gradient
(– 4, 256)	– 4	**– 256**
(– 3, 81)	– 3	**– 108**
(– 2, 16)	– 2	**– 32**
(– 1, 1)	– 1	**– 4**
(0, 0)	0	**0**
(1, 1)	1	**4**
(2, 16)	2	**32**
(3, 81)	3	**108**
(4, 256)	4	**256**

Gradient = $\mathbf{4x^3}$

Equation of Curve	Gradient function
$y = x^2$	$2x$
$y = x^3$	$\mathbf{3x^2}$
$y = x^4$	$\mathbf{4x^3}$
$y = x^5$	$\mathbf{5x^4}$